Primordial magnetic fields and CP violation in the sky

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It has been argued that electroweak baryogenesis generates a left-handed primordial magnetic field. The helicity density of the primordial magnetic field today is estimated to be $\sim 10^2 n_b$ where $n_b \sim 10^{-6}$ /cm^3 is the present cosmological baryon number density. With certain assumptions about the inverse cascade the field strength at recombination is $\sim 10^{-13}$ G on a coherence scale $\sim 10^{-4}$ pc. Here I discuss the various assumptions made in obtaining these estimates and the concomitant open issues.

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The observed galactic and cluster magnetic fields of strength $\sim 10^{-6}$ Gauss could be due to amplification of a primordial seed field. Hence it is of interest to find early universe scenarios that can lead to primordial seed fields. If the galactic dynamo is maximally efficient, the seed field strength should be larger than 10^{-21} Gauss at the proto-galaxy stage of structure formation. Another characteristic of the observed magnetic fields is their large coherence scale. The "homogeneous" component of the galactic magnetic field has a typical coherence scale ~ 1 kpc. Assuming that the coherence scale cannot grow during galaxy evolution, the coherence scale of the seed field should also be ~ 1 kpc at the present epoch.

1 Magnetic field production

In Ref. [1] I pointed out that electroweak baryogenesis would imply a primordial magnetic field (also see [2]). The relationship between baryogenesis and magnetic fields follows once it is realized that baryon number violation proceeds via twisted non-Abelian gauge fields such as the sphaleron or electroweak string configurations. These gauge field configurations are the intermediate step in processes that change baryon number. The present baryonic excess is produced as these configurations decay. The argument in Ref. [1] is that the decay also produces an electromagnetic magnetic field, $\mathbf{B}$, that carries some of the twist in the non-Abelian gauge fields making up the configuration. An estimate of the helicity ($\mathcal{H}$) of $\mathbf{B}$ yields

$$\mathcal{H} \equiv \frac{1}{V}\int_V d^3x \mathbf{A}\cdot\mathbf{B} \sim -\frac{n_b}{\alpha} \tag{1}$$

where $\alpha = 1/137$ is the fine structure constant and n_b is the baryon density of the universe. The sign of the helicity density is determined by the sign of the baryon number. Since we observe baryons and not anti-baryons, the primordial magnetic field has negative helicity density *i.e.* it is left-handed everywhere.

The estimate in eq. (1) is based on a specific decay channel of a certain configuration of electroweak strings, namely, two linked loops of Z-string. The estimate in eq. (1) assumes that other decay channels also lead to roughly the same helicity. Testing this assumption is the first open issue.

Issue 1: *Can one quantify more accurately the* $\mathbf{B}$ *helicity produced by baryon number violating processes?*

In principle one could imagine watching a sphaleron decay on a computer and then tracking the $\mathbf{B}$ helicity. This would provide a check of the above estimate for another decay channel and would be very valuable. However, the sphaleron would first have to be constructed for non-zero weak mixing angle (θ_w), perturbed, and then studied. This is a non-trivial numerical task since one would like to evolve all of the electroweak bosonic degrees of freedom. Besides, for the purpose of the present application, we are

not interested in the detailed dynamics of all the bosonic degrees of freedom. We only want to know how the twist (Chern-Simons number) in the sphaleron gauge fields gets transferred to the **B** field helicity. I have a feeling that it should be possible to extract the latter without solving for the whole evolution. Whether this can be done is a very interesting and important question.

2 Magnetic field evolution

Once helical magnetic fields are produced, the evolution is described by the MHD equations in an expanding spacetime. In the case of a flat geometry, it can be shown that the field variables can be rescaled so that the evolution is given by the MHD equations in a non-expanding spacetime [3]. The initial **B** field will be produced on very small length scales. How does such a magnetic field evolve?

A magnetic field coherent only on very small length scales will simply dissipate. This follows from the MHD equation

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}) + \frac{1}{4\pi\sigma_c}\nabla^2 \mathbf{B} \tag{2}$$

where $\mathbf{v}$ is the plasma velocity and σ_c the electrical conductivity. The second term on the right-hand side causes dissipation and is proportional to $1/L^2$ where L is the coherence scale of the field. Therefore when L is very small the second term greatly dominates the first term on the right-hand side and the field decays exponentially fast. In the opposite situation, where the first term greatly dominates the second term (i.e. for large coherence scales and velocity flows) it can be shown that the magnetic field is frozen-in the plasma and simply gets dragged with the flow of the plasma. The condition under which the first term dominates over the second term is that the magnetic Reynold's number $\mathcal{R}_M$ be large:

$$\mathcal{R}_M \equiv 4\pi\sigma_c L v >> 1 \ \Rightarrow \ v >> \frac{e^4}{4\pi} \tag{3}$$

This raises the second open issue.

Issue 2: *What are the plasma velocities implied by electroweak baryogenesis? Are they large enough to satisfy the condition in eq. (3)?*

There are several different scenarios for successful electroweak baryogenesis. All of them satisfy the Sakharov condition that thermal equilibrium should be violated during the epoch of baryogenesis. This is usually achieved by considering a strongly first order phase transition with typical bubble wall velocities ~ 0.1 (for a review see [4]). In such scenarios the fluid velocities are clearly large enough and the first term in the MHD equation is dominant; yet the second term cannot be neglected. This brings us to issue 2 – a rather general issue involving cosmological phase transitions and MHD.

Issue 3: *How does the magnetic field helicity evolve during and after the electroweak phase transition?*

There are some hints regarding this issue that are already present in the MHD literature in connection with the "reversed magnetic pinch". If a plasma is confined to a cylindrical volume and an axial current is applied together with a uniform axial magnetic field, it is found that the plasma relaxes in such a way that the final magnetic field at the surface of the cylinder is anti-parallel to that on the cylinder's axis. The phenomenon was explained by J.B. Taylor [5] by hypothesizing that magnetic helicity in an infinite volume ($V \to \infty$ in eq. (1)) is conserved even when the second term in the MHD equation is non-zero provided it is sub-dominant as compared to the first term. If we adopt this hypothesis, it would imply that, regardless of the details of the evolution during the electroweak phase transition, the global magnetic helicity will be conserved. This is a very important constraint on the evolution and, as we shall see, will eventually allow us to estimate the magnetic field strength.

3 Inverse cascade

The coherence scale of the magnetic field generated at the electroweak phase transition is microphysical. If only the Hubble expansion were to stretch the coherence scale, this would amount to a coherence scale of about 1 centimeter today. However, astrophysical magnetic fields are coherent on kpc scales. Is there a way to bridge this enormous gap?

There is evidence in the MHD literature that under certain circumstances a magnetic field can "inverse cascade". A direct cascade is when energy is transferred from larger to smaller scales, where it is ultimately dissipated. An inverse cascade is the transfer of energy from short to long wavelengths. It is believed that the inverse cascade can occur provided the plasma is turbulent and the magnetic field is helical. As discussed above, both these ingredients are present during the electroweak phase transition. Hence we can expect an inverse cascade that will stretch the coherence scale of the magnetic field.

The growth of the coherence scale due to Hubble expansion can be factored in quite trivially and so let us consider MHD in a non-expanding spacetime. If $L(t)$ is the time-dependent coherence scale, we can write

$$L(t) = L(t_i)\left(\frac{t}{t_i}\right)^p \tag{4}$$

where, t_i is some initial time, and p is the inverse cascade exponent. At present, there is some debate over the value of p. Renormalization group calculations indicate that $p = 0$ [6], analytical analyses point to $p = 2/3$ [7], while numerical simulations give $p \sim 1/2$ [8, 9] The value of p is quite important and the estimates in [1] take $p = 2/3$. This brings us to the next issue.

Issue 4: *What is the value of p and how does it affect the cosmic evolution of the primordial magnetic field?*

This issue is further complicated by the occurrence of certain cosmic episodes. For example, when electron-positron annihilation takes place, the electrical conductivity of the cosmological plasma suddenly drops and the second term in the MHD equation gains importance. If $p = 1/2$, it turns out that the coherence scale of the magnetic field has not grown sufficiently that it can survive the larger dissipation in the post e^+e^- era. In this case, only certain large wavelength modes of the magnetic field can survive. The field strength due to these modes will be smaller. However the analysis of Ref. [1] does not yield the spectrum of the generated magnetic field and so we cannot estimate this field strength. This is issue 5.

Issue 5: *What is the spectrum of the magnetic field resulting from electroweak baryogenesis?*

Another cosmic episode that may have an effect on the primordial helical magnetic field is the QCD phase transition. Depending on the nature of the phase transition, there may be turbulence in the plasma and energy may be injected into the helical magnetic field. This would be worth investigating further together with the impact of other cosmic events on helical magnetic fields.

Issue 6: *What effect does the QCD phase transition and other cosmic events have on pre-existing helical magnetic fields?*

Assuming $p = 2/3$, it is easy to estimate the coherence scale of the magnetic field. This turns out to be $\sim 10^{14}$ cms at recombination. Once we have the coherence scale, we can use the conservation of magnetic helicity, $\sim B^2 L \sim n_b/\alpha$, to estimate the field strength: $B(t_{rec}) \sim 10^{-13}$ Gauss.

The coherence scale corresponds to ~ 0.1 pc today and is smaller than the observed kpc scale by 10^4. However, the field strength is well within what a galactic dynamo can (in principle) amplify to produce the presently observed field strengths of $\sim 10^{-6}$ Gauss. The primordial field will certainly have some power on kpc scales but at present we cannot say if it will be sufficient for the dynamo. That will depend on the spectrum of the field produced during baryogenesis (Issue 5).

4 CP violation in the sky

Finally I would like to discuss the prospects for observing a helical magnetic field. Several papers have discussed the observation of non-helical magnetic fields using the cosmic microwave background radiation (CMBR) but have missed discussing helical fields. The reason is that they have taken the correlators of the magnetic field Fourier coefficients to satisfy

$$\langle b_i^*(\mathbf{k}) b_j(\mathbf{k}) \rangle \propto \left(\delta_{ij} - \frac{k_i k_j}{k^2} \right) \tag{5}$$

where $\mathbf{k}$ labels Fourier modes, and δ_{ij} is the Kronecker delta function. Note that the right-hand side is symmetric in i, j and this means that the magnetic field is non-helical,

that is $\langle \mathbf{B} \cdot \nabla \times \mathbf{B} \rangle = 0$. Hence, we are interested in the case when the above correlator has an extra piece that is anti-symmetric in i, j, namely proportional to $\epsilon_{ijl} k_l$.

The way that a non-helical magnetic field shows up in the CMBR is that it exerts a Lorentz force on the plasma and causes fluid flow. The additional velocity of the plasma can be detected in the photons due to the Doppler effect. This scheme fails for helical fields since the helical part (the anti-symmetric contribution to the correlator in eq. (5)) is "force-free" *i.e.* does not exert any Lorentz force on the plasma! There is a higher order effect though which is discussed in [10], due to the different scattering cross-sections of photons off of electrons and protons. The effect causes non-vanishing correlations between the CMBR temperature and odd parity polarization (C_l^{TB}) and also different types of polarization (C_l^{EB}). However the magnitude of these correlators is tiny and hence it does not seem likely that the helicity of the magnetic field can be detected in the CMBR. However, there might be other effects that have not been considered and this is the next issue.

Issue 7: *Can helical magnetic fields be detected in the CMBR? Or more generally, can the helicity of a cosmological/astrophysical magnetic field be detected by any means?*

The detection of a helical cosmic magnetic field would be a direct detection of CP violation in cosmology. In principle, the observation of such magnetic fields can lead to a probe of the electroweak phase transition and the intervening cosmic history. This would be in addition to providing a better understanding of the observed magnetic fields in galaxies and clusters of galaxies.

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